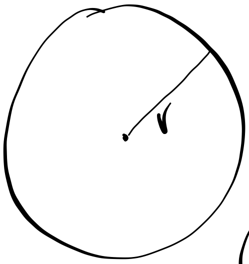
3.11/9 ① day 46



② $A(t) = \pi \, r(t)^2$
(true at all times t)

Know $\frac{dA}{dt} = +1 \, \frac{cm^2}{sec}$

③ $\boxed{\frac{dA}{dt} = \pi\left(2r\frac{dr}{dt}\right)}$
(true at all times t)

4a) when r = 2cm

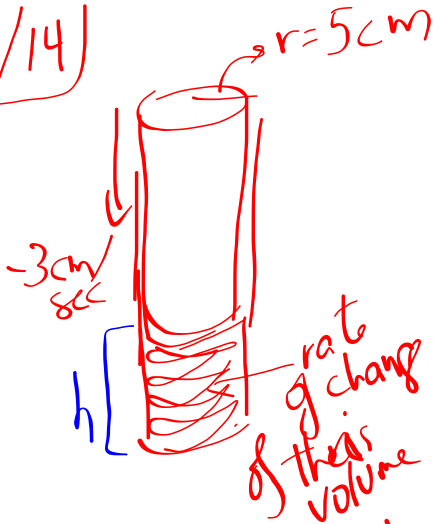
$1 = \pi\left(2(2)\frac{dr}{dt}\right)$ so $\frac{dr}{dt} = \frac{1}{4\pi} \frac{cm}{sec}$

4b) when $C = 2\pi r = 2cm \Rightarrow r = \frac{1}{\pi} cm$

$\frac{dA}{dt} = 1 = \pi\left(2\left(\frac{1}{\pi}\right)\frac{dr}{dt}\right) \Rightarrow \frac{dr}{dt} = \frac{1}{2} \frac{cm}{sec}$

3.11/14

day 46



2) $V=\pi r^2 h$ $\qquad \frac{dV}{dt}=\pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$

$V=25\pi h$

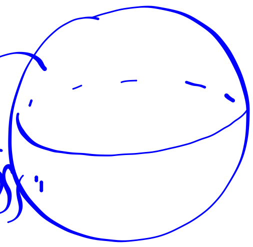
3) $\frac{dV}{dt}=25\pi\frac{dh}{dt}$

4) $\frac{dV}{dt}=25\pi(-3)=-75\pi\ \frac{cm^3}{sec}$

day 46

3.11/15)

'I'm melting's'



2) $V = \frac{4}{3}\pi r^3$ $\qquad SA = 4\pi r^2$

3) $\frac{dV}{dt} = \frac{4\pi}{3}\left(3r^2\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

(4) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = k(4\pi r^2)$

so $\frac{dr}{dt} = k$

Nov 4-7:30 AM

day 46

3.7/101

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

⋮

def^{n}s of derivative

$$\lim_{x \to 5} \frac{f(x^2) - f(25)}{x - 5}$$

5^2!

$f(x) \approx f(x^2)$

$a \approx 5$

$$= \frac{d}{dx}\left(f(x^2)\right)\Big|_{x=5}$$

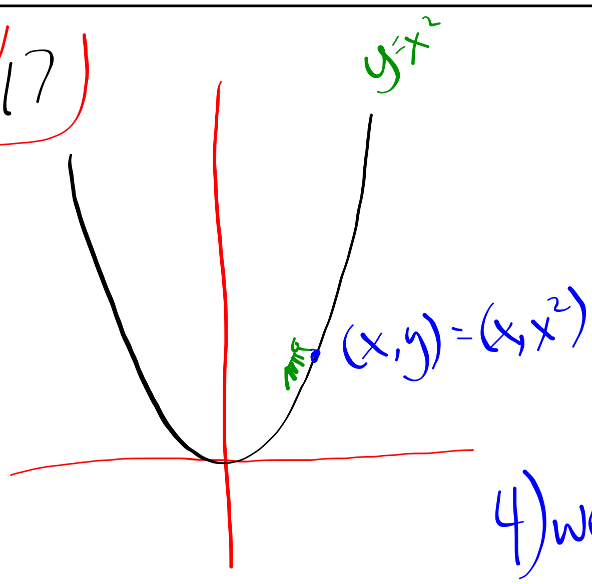
$$= f'(x^2) \cdot (2x)\Big|_{x=5} = f'(25) \cdot 10$$

$$= 10 f'(25)$$

Nov 4-7:30 AM

day 46

3.11/17



(2) $y = x^2$

(3) $\frac{dy}{dt} = 2x\frac{dx}{dt}$

4) we want $\frac{dy}{dt} = \frac{dx}{dt}$

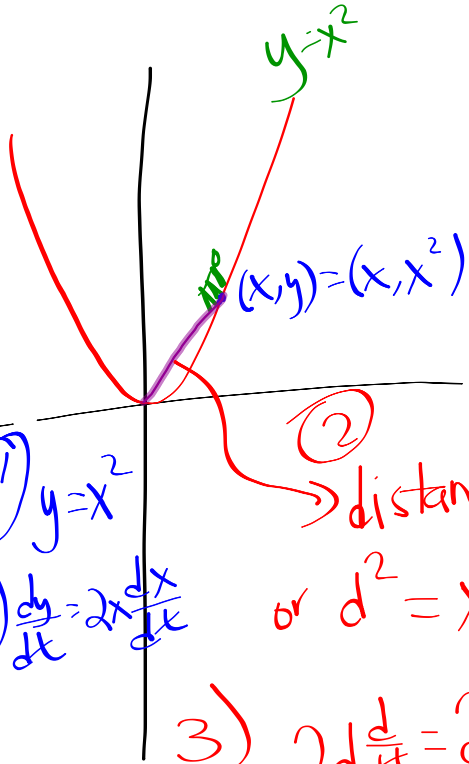
so $\frac{dx}{dt} = 2x\frac{dx}{dt} \Rightarrow 1 = 2x$

or

$x = \frac{1}{2}, y = \frac{1}{4}$

day 46

3.11/actual 16)



Know

$\frac{dd}{dt}=+1\ \frac{cm}{min}$

$Pt=(2,4)$

$d=\sqrt{2^2+4^2}$

$d=\sqrt{20}$

(2) $\text{distance}=\sqrt{(x)^2+(y)^2}$

or $d^2=x^2+(x^2)^2=x^2+x^4$

3) $2d\frac{dd}{dt}=2x\frac{dx}{dt}+4x^3\frac{dx}{dt}$

or $d\ \frac{dd}{dt}=\frac{dx}{dt}(x+2x^3)$

(2') $y=x^2$

(3') $\frac{dy}{dt}=2x\frac{dx}{dt}$

(4) $\sqrt{20}\,(1)=\frac{dx}{dt}(2+2(2^3))=$

$\frac{dx}{dt}=\frac{\sqrt{20}}{18}\ cm/min$

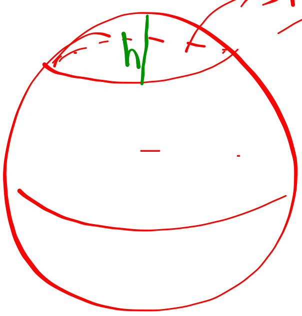
(4') $\frac{dy}{dt}=2(2)\left(\frac{\sqrt{20}}{18}\frac{cm}{min}\right)$

$=\frac{2\sqrt{20}}{9}\ \frac{cm}{min}$

Nov 4-7:30 AM

day 46

3.8/75)



spherical cap

$$\star \quad V = \frac{\pi h^2(3r-h)}{3}$$

a) find $\frac{dr}{dh}$ when $V_{sphere} = \frac{5\pi}{3}$

take derivative of $\star$ wrt h

$$0 = \frac{dV}{dh} = \frac{\pi}{3}\left[2h(3r-h) + (h^2)\left(3\frac{dr}{dh} - 1\right)\right]$$

$V = \frac{4}{3}\pi r^3 = \frac{5\pi}{3}$,

$4r^3 = 5$

$r = \sqrt[3]{5/4}$

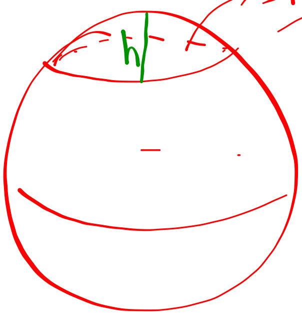
$$0 = \frac{\pi}{3}\left[2h\left(3\sqrt[3]{\tfrac{5}{4}} - h\right) + h^2\left(3\frac{dr}{dh}\right) - h^2\right.$$

$$\frac{-2h\left(3\sqrt[3]{\frac{5}{4}} - h\right) + h^2}{3h^2} = \frac{dr}{dh}$$

B.8/75)

day 46

spherical cap



$$V = \frac{\pi h^2(3r-h)}{3}$$

$$\frac{5\pi}{3} = \frac{\pi h^2(3r-h)}{3}$$

2) $5\pi = \pi h^2(3r-h)$

$5 = h^2(3r-h)$

3) $\frac{d}{dh}(5)$ $\quad 0 = 2h(3r-h) + h^2\left[\left(3\frac{dr}{dh}\right) - 1\right]$

$-6rh + 2h^2 + h^2 = 3h^2 \frac{dr}{dh}$

$\Rightarrow \boxed{-2r + h = h\frac{dr}{dh}}$

$$\frac{-2r+h}{h} = \frac{dr}{dh}$$

Nov 4-7:30 AM

day 46

3.9/16)

$$\frac{d}{dx}\left(e^x \ln x\right)$$

PRODUCT

NOT COMPOSITION!

$$\frac{d}{dx}(e^x)\cdot \ln x + e^x \cdot \frac{d}{dx}(\ln x)$$

$$= e^x \ln x + \frac{e^x}{x}$$

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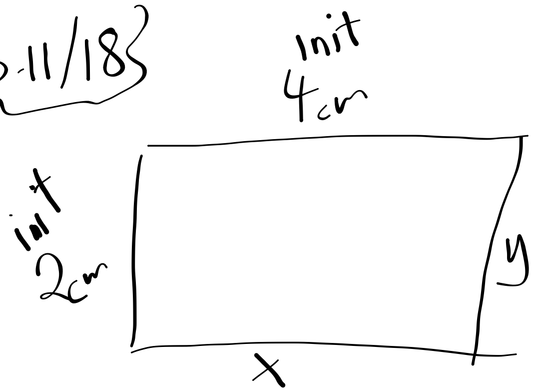
3.11/18

day 46



$\frac{dx}{dt} = +1$ cm/sec

$\frac{dy}{dt} = +1$ cm/sec

② $A = xy$

③ $\frac{dA}{dt} = \frac{dx}{dt} \cdot y + x \frac{dy}{dt}$

after 20 sec

$x = 4 + 20(+1) = 24$ cm

$y = 2 + 20(+1) = 22$ cm

So

④ $\frac{dA}{dt} = (1)(22) + (1)(24) = 46 \frac{cm^2}{sec}$

Nov 4-7:30 AM